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Plane Symmetric Chaplygin Gas Dark Energy Model in *f(T)* Theory of Gravity

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Abstract:

The plane symmetric universe filled with Chaplygin gas (dark energy) has been studied in $f(T)$ theory of gravity. Detail study has been carried out on the equation of state parameter and phantom divide line .

Keywords:*Plane Symmetric Space Time, $f(T)$ theory of gravity, Equation of state parameter, Chaplygin gas , Dark energy.*

[1] Introduction :

The Dark Energy models are obtained by modifying the matter component of the universe without changing the gravitational part. Some of the well known Dark Energy models are designed using cosmological constant, quintessence, K'essence and perfect fluid as a matter (Kamenshchik *et al.* 2001; Li 2004; Cai 2007; Wei 2009; Sheykhi & Jamil 2011). The modified theories of gravity depend upon modifying the gravitational part of the Einstein-Hilbert action and keeping the matter component unchanged. The most studied modified theories of gravity are $f(R)$, $f(R,T)$, $f(T,\tau)$, $f(R,L_m)$ and so on. Interesting results have been found in the *f(T)* theory of gravity. Ferraro and Fiorini (2007, 2008) developed this theory for solving particle horizon problem and obtained singularity free solutions with positive cosmological constant. Bamba *et al*. (2011) have studied the evolution of effective equation of state for different *f(T)* models. Daouda *et al*.(2012) considered this model with the holographic Dark Energy model and found that the reconstructed model shows phantom behavior as well as unification of Dark Energy and Dark Matter. In this paper Plane symmetric Chaplygin Gas Dark Energy model in *f(T)* theory of gravity has been studied .

[2] $f(T)$ gravity field equations :

The metric tensor $g_{\mu\nu}$ is given by the following relation

$$g_{\mu\nu} = \eta_{ij} h^i_\mu h^j_\nu , \qquad (1)$$

where $\eta_{ij} = diag(1,-1,-1,-1)$ is the Minkowski metric for the tangent space.

For a given metric there exist infinite different tetrad fields h^i_μ which satisfy the following properties (Ferraro and Fiorini 2007)

$$h^i_\mu h^\mu_j = \delta^i_j , \quad h^i_\mu h^\nu_i = \delta^\nu_\mu . \qquad (2)$$

The action in a Universe governed by *f(T)* gravity is given by

$$I = \frac{1}{16\pi G}\int d^4xe(T + f(T) + L_m) ,$$

where G is the gravitational constant and $e = \sqrt{-g}$, L_m stands for the matter Lagrangian and $f(T)$ is a general differentiable function of torsion.

The teleparallel Lagrangian density is described by the torsion scalar T and is defined as

$$T = S_\rho{}^{\mu\nu} T^\rho{}_{\mu\nu}, \qquad (3)$$

where

$$S_\rho{}^{\mu\nu} = \frac{1}{2}\left(K^{\mu\nu}{}_\rho + \delta^\mu_\rho T^{\theta\nu}{}_\theta - \delta^\nu_\rho T^{\theta\mu}{}_\theta\right), \quad (4)$$

is the antisymmetric tensor.

The torsion and contorsion tensors are defined as

$$T^\rho{}_{\mu\nu} = \Gamma^\rho{}_{\nu\mu} - \Gamma^\rho{}_{\mu\nu} = h^\rho_i(\partial_\mu h^i_\nu - \partial_\nu h^i_\mu) , \quad (5)$$

$$K^{\mu\nu}{}_{\rho} = -\frac{1}{2}\left(T^{\mu\nu}{}_{\rho} - T^{\nu\mu}{}_{\rho} - T_{\rho}{}^{\mu\nu}\right). \qquad (6)$$

The modified field equations of the teleparallel theory of gravity (Sharif and Jawad 2012) are obtained by varying the action with respect to the vierbein vector field h^{i}_{μ} and are given by

$$\left[e^{-1}\partial_{\mu}\left(eS_{i}{}^{\mu\nu}\right) - h_{i}^{\lambda}T^{\rho}{}_{\mu\lambda}S_{\rho}{}^{\nu\mu}\right]\left(1+f(T)\right) + S_{i}{}^{\mu\nu}\partial_{\mu}(T)f_{TT} + \frac{1}{4}h_{i}^{\nu}\left[T+f(T)\right] = \frac{1}{2}k^{2}h_{i}^{\rho}T_{\rho}^{\nu}, \qquad (7)$$

Where $S_{i}^{\mu\nu} = h_{i}^{\rho}S_{\rho}^{\mu\nu}$, $k^{2} = 8\pi G$, $f_{T} \equiv \frac{df}{dT}$.

[3] Plane Symmetric Cosmological Solutions :

The line element in plane symmetric form has been written as

$$ds^{2} = dt^{2} - A^{2}(t)(dx^{2}+dy^{2}) - B^{2}(t)dz^{2}, \qquad (8)$$

where A and B are the cosmic scale factors.

Using equations (1) and (8), the tetrad components are obtained as follows

$$h^{i}_{\mu} = diag(1, A, A, B),$$

$$h_{i}^{\mu} = diag(1, A^{-1}, A^{-1}, B^{-1}). \qquad (9)$$

After substituting Equations (4) and (5) in equation (3) the torsion T becomes

$$T = -2\left(2\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}^{2}}{A^{2}}\right). \qquad (10)$$

The average scale factor a, the mean Hubble parameter H and the anisotropy parameter Δ of the expansion respectively (for plane symmetric metric) becomes

$$a = (A^{2}B)^{\frac{1}{3}}, \qquad (11a)$$

$$H = \frac{1}{3}\left(2\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right), \qquad (11b)$$

$$\Delta = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_{i}-H}{H}\right)^{2}, \qquad (11c)$$

where H_{i} are the directional Hubble paramerters in x, y and z direction respectively which are given as $H_{1} = H_{2} = \frac{\dot{A}}{A}$, $H_{3} = \frac{\dot{B}}{B}$.

For perfect fluid, the energy momentum tensor is defined as

$$T_{\rho}^{\nu} = diag(\rho_{M}, -P_{M}, -P_{M}, -P_{M}), \qquad (12)$$

where ρ_{M} is the energy density and P_{M} is the pressure of matter inside the Universe.

Using equations (8-12), the field equations (7) of *f(T)* gravity for $i = 0 = \nu$ and $i = 3 = \nu$ reduce to following set of equations:

$$T + f(T) - 4\left(2\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}^{2}}{A^{2}}\right)\left(1+f(T)\right) = 2k^{2}\rho_{M}, \qquad (13)$$

$$4\left(\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}^{2}}{A^{2}} + \frac{\ddot{A}}{A}\right)\left(1+f(T)\right) - 16\frac{\dot{A}}{A}\left[\frac{\dot{A}}{A}\left(\frac{\ddot{B}}{B} - \frac{\dot{B}^{2}}{B^{2}}\right) + \left(\frac{\ddot{A}}{A} - \frac{\dot{A}^{2}}{A^{2}}\right)\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right)\right]f_{TT} - (T+f) = 2k^{2}P_{M} \qquad (14)$$

For solving the field equations (13) &(14), we consider that the expansion θ is proportional to shear σ. {Thorne 1967 suggests that from the observations of velocity red shift relation for extragalactic sources have shown that the isotropy of the Hubble expansion of the universe holds within about 30% range approximately (Kantowski and Sachs 1966) and further studies have given the limit to be $\frac{\sigma}{H} \leq 0.30$, where σ is shear and H is Hubble constant. }.

The expansion scalar and shear scalar (plane symmetric universe) is given by

$$\theta = 2\frac{\dot{A}}{A} + \frac{\dot{B}}{B}, \qquad (15)$$

$$\sigma = \frac{1}{\sqrt{3}}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right). \qquad (16)$$

It leads to the condition (Bali and Kumavat 2008)

$$B(t) = A^{m}(t), \quad m \geq 2. \qquad (17)$$

Using this condition, we found

$$\Delta = 2\frac{(m-1)^{2}}{(m+2)^{2}}. \qquad (18)$$

For Δ=0 , here , the isotropic behavior of the expanding Universe will be obtained.

Using this condition above field equation becomes,

$$H^{2} = -\frac{(m+2)^{2}}{9(2m+1)}\left(\frac{8\pi G}{3}\rho_{M} - \frac{f}{6} - \frac{Tf_{T}}{3}\right). \qquad (19)$$

$$\left(H^{2}\right)' = \frac{m+2}{3}\left(\frac{2k^{2}P_{M} + \frac{2(m+2)}{2m+1}Tf_{T} + \frac{4m+5}{2m+1}T + f}{2+2f_{T}+4Tf_{TT}}\right). \qquad (20)$$

where prime denotes the derivative with respect to $\ln\left(A^{\frac{m+2}{3}}\right)$.

[4] Standard Chaplygin Gas Dark Energy Cosmological Model:

Here, we choose the scale factor of the form

$B(t)=A^m(t)=a(t)=a_0(t_s-t)^{-h}$ $\quad ,t\le t_s,\quad m\ge 2\quad ,h>0$,(21)

where n and s are positive real constants. For convenience , we take $s=1$.

The equation of state of the standard chaplygin gas dark energy is given by

$$p_\gamma=\frac{-C}{\rho_\gamma}\quad , \qquad (22)$$

where $C>0$ positive constant .

For $T+f(T)=T$, the field equations (13) , (14) , (19) and (20) reduce to

$$H^2=\frac{(m+2)^2}{9(2m+1)}\left[-\frac{8\pi G}{3}(\rho_M+\rho_{DE})\right]. \quad (23)$$

$$(H^2)'=\left(\frac{m+2}{3}\right)8\pi G\left[P_M+P_{DE}+\frac{4m+5}{3(2m+1)}(\rho_M+\rho_{DE})\right]$$

. (24)

Here we have assumed (for non-relativistic matter) $P_M=0$.

Using above all equations, the energy density and pressure of Standard Chaplygin Gas Dark Energy becomes

$$\rho_{DE}=\frac{-1}{16\pi G}(f+2Tf_T). \qquad (25)$$

$$P_{DE}=\frac{1}{16\pi G}\left(\frac{f-Tf_T-\frac{2(4m+5)}{2m+1}T^2f_{TT}}{1+f_T+2Tf_{TT}}\right). \quad (26)$$

The energy conservation equations corresponding to standard matter and Dark Energy are

$$\dot\rho_M+3H\rho_M=0,$$

$$\dot\rho_{DE}+3H\rho_{DE}(1+\omega_{DE})=0 \qquad (27)$$

Here $\omega_{DE}=\frac{P_{DE}}{\rho_{DE}}$ is the Equation of State parameter for standard chaplygin gas dark energy in $f(T)$ gravity.

The resulting Equation of State parameter for standard chaplygin gas dark energy in $f(T)$ gravity will be given by

$$\omega_{DE}=-\frac{\frac{f}{T}-f_T-\frac{2(4m+5)}{2m+1}Tf_{TT}}{(1+f_T+2Tf_{TT})\left(\frac{f}{T}+2f_T\right)}=-1+\frac{1}{\frac{C}{D}\left[a_0\left(\frac{H}{h}\right)^h\right]^6+1},\quad h>0 .$$

(28)

For $D<0$, ω_{DE} can cross the phantom divide line.

[5]Conclusion

The Standard Chaplygin Gas Dark Energy model in *f(T)* gravity for Plane Symmetric universe has been studied. It is interesting to note that the Equation of State parameter ω_{DE} can cross the phantom divide line in *f(T)* theory of gravity .

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