Plane Symmetric Polytropic Gas Dark Energy Model in *f(T)* Theory of Gravity

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Abstract:

The plane symmetric universe filled with polytropic gas dark energy has been studied in $f(T)$ theory of gravity. The equation of state parameter of the $f(T)$ gravity model has been derived . The necessary conditions for crossing the phantom-divide line has been obtained

Keywords:*Plane Symmetric Universe, $f(T)$ theory of gravity, Equation of state parameter, Polytropic gas, Dark energy.*

1] Introduction :

The modified theories of gravitation are $f(R)$, $f(R,T)$, $f(T,\tau)$, $f(R,L_m)$ and so on (Nojiri & Odintsov 2007, Sotiriou & Faraoni 2010, Felice & Tsujikawa 2010, Houndjo 2012, Sadeghi *et al*. 2013, Harko *et al*. 2014). Amongst these modified theories interesting results have been found in the *f(T)* theory of gravity which is the generalization of teleparallel gravity. Bianchi type I universe has been investigated by Sharif and Rani (2011a) and discussed the accelerated expansion of the universe. Daouda *et al*.(2012) studied $f(T)$ models with the holographic Dark Energy . Linder (2010), Sharif and Rani (2011b), Sharif and Azeem (2013), Sharif and Nazir (2015) have studied several *f(T)* models in different contexts.Above investigations motivated me to examine $f(T)$ model by using plane symmetric Universe.

2] $f(T)$ gravity formalism :

The orthogonal tetrad components $h_i(x^\mu)$, where Latin as well as Greek indices takes the values $0,1,2,3$ which forms an orthogonal basis for the tangent space at each point x^μ of the manifold, have been used in teleparallelism.

The metric tensor $g_{\mu\nu}$ is given by the following relation

$$g_{\mu\nu} = \eta_{ij} h^i_\mu h^j_\nu , \qquad (2.1)$$

where $\eta_{ij} = diag(1,-1,-1,-1)$ is the Minkowski metric for the tangent space.

For a given metric there exist infinite different tetrad fields h^i_μ which satisfy the following properties (Ferraro and Fiorini 2007).

$$h^i_\mu h^\mu_j = \delta^i_j , \quad h^i_\mu h^\nu_i = \delta^\nu_\mu . \qquad (2.2)$$

The action in a Universe governed by *f(T)* gravity is given by

$$I = \frac{1}{16\pi G}\int d^4xe(T + f(T) + L_m) ,$$

where G is the gravitational constant and $e = \sqrt{-g}$, L_m stands for the matter Lagrangian and $f(T)$ is a general differentiable function of torsion.

The teleparallel Lagrangian density is described by the torsion scalar T and is defined as

$$T = S_\rho{}^{\mu\nu} T^\rho{}_{\mu\nu}, \qquad (2.3)$$

where

$$S_\rho{}^{\mu\nu} = \frac{1}{2}\left(K^{\mu\nu}{}_\rho + \delta^\mu_\rho T^{\theta\nu}{}_\theta - \delta^\nu_\rho T^{\theta\mu}{}_\theta\right), \qquad (2.4)$$

is the antisymmetric tensor.

The torsion and contorsion tensors are defined as

$$T^\rho{}_{\mu\nu} = \Gamma^\rho{}_{\nu\mu} - \Gamma^\rho{}_{\mu\nu} = h^\rho_i(\partial_\mu h^i_\nu - \partial_\nu h^i_\mu) , \qquad (2.5)$$

$$K^{\mu\nu}{}_{\rho} = -\frac{1}{2}\left(T^{\mu\nu}{}_{\rho} - T^{\nu\mu}{}_{\rho} - T_{\rho}{}^{\mu\nu}\right). \qquad (2.6)$$

The modified field equations of the teleparallel theory of gravity (Sharif and Jawad 2011) are obtained by varying the action with respect to the vierbein vector field h^{i}_{μ} as (Bengochea and Ferraro 2009)

$$\left[e^{-1}\partial_{\mu}\left(eS_{i}{}^{\mu\nu}\right) - h_{i}^{\lambda}T^{\rho}{}_{\mu\lambda}S_{\rho}{}^{\nu\mu}\right]\left(1+f(T)\right) + S_{i}{}^{\mu\nu}\partial_{\mu}(T)f_{TT} + \frac{1}{4}h_{i}^{\nu}\left[T+f(T)\right] = \frac{1}{2}k^{2}h_{i}^{\rho}T_{\rho}^{\nu},$$

Where $S_{i}^{\mu\nu} = h_{i}^{\rho}S_{\rho}^{\mu\nu}$, $k^{2} = 8\pi G$, $f_{T} \equiv \frac{df}{dT}$.
(2.7)

[3] Plane Symmetric Cosmological Solutions :

Considering the line element [Zhang & Noh 2009, Setare & Momeni 2010, Rao *et al*. 2010, Shen & Zhao 2012] in plane symmetric form as

$$ds^{2} = dt^{2} - A^{2}(t)(dx^{2} + dy^{2}) - B^{2}(t)dz^{2}, \qquad (3.1)$$

where A and B are the cosmic scale factors.

Using equations (2.1) and (3.1), the tetrad components are obtained as follows

$$h^{i}_{\mu} = diag(1, A, A, B),$$

$$h_{i}^{\mu} = diag(1, A^{-1}, A^{-1}, B^{-1}). \qquad (3.2)$$

After substituting equations (2.4) and (2.5) in equation (2.3) the torsion T reduces to

$$T = -2\left(2\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}^{2}}{A^{2}}\right). \qquad (3.3)$$

The average scale factor a, the mean Hubble parameter H and the anisotropy parameter Δ of the expansion respectively becomes

$$a = (A^{2}B)^{\frac{1}{3}}, \qquad (3.4)$$

$$H = \frac{1}{3}\left(2\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right), \qquad (3.5)$$

$$\Delta = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_{i} - H}{H}\right)^{2}, \qquad (3.6)$$

where H_{i} are the directional Hubble paramerters in x, y and z direction respectively which are given by $H_{1} = H_{2} = \frac{\dot{A}}{A}$, $H_{3} = \frac{\dot{B}}{B}$.

For perfect fluid the energy momentum tensor is defined as

$$T_{\rho}^{\nu} = diag(\rho_{M}, -P_{M}, -P_{M}, -P_{M}), \qquad (3.7)$$

where ρ_{M} is the energy density and P_{M} is the pressure of matter inside the Universe.

Using equations (3.1 to 3.7), the field equations (2.7) of *f(T)* gravity for $i = 0 = \nu$ and $i = 3 = \nu$ reduce to following set of equations:

$$T + f(T) - 4\left(2\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}^{2}}{A^{2}}\right)\left(1 + f(T)\right) = 2k^{2}\rho_{M}, \qquad (3.8)$$

$$4\left(\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{A}^{2}}{A^{2}} + \frac{\ddot{A}}{A}\right)\left(1+f(T)\right) - 16\frac{\dot{A}}{A}\left[\frac{\dot{A}}{A}\left(\frac{\ddot{B}}{B} - \frac{\dot{B}^{2}}{B^{2}}\right) + \left(\frac{\ddot{A}}{A} - \frac{\dot{A}^{2}}{A^{2}}\right)\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right)\right]f_{TT} - \left(T + f\right) = 2k^{2}P_{M} \qquad (3.9)$$

To solve the field equations (3.8) &(3.9), we consider that the expansion θ is proportional to shear σ. {please refer Kantowski and Sachs 1966 Thorne 1967}.

Hence, (For Plane Symmetric Universe) the expansion scalar and shear scalar is given by

$$\theta = 2\frac{\dot{A}}{A} + \frac{\dot{B}}{B}, \qquad (3.10)$$

$$\sigma = \frac{1}{\sqrt{3}}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right). \qquad (3.11)$$

It leads to the condition (Bali and Kumavat 2008; Bali and Singh 1987; Amirhashchi 2011)

$$B(t) = A^{m}(t), \quad m \geq 2. \qquad (3.12)$$

Using this condition, we found

$$\Delta = 2\frac{(m-1)^{2}}{(m+2)^{2}}. \qquad (3.13)$$

For Δ=0 we have obtained here the isotropic behavior of the expanding Universe.

Using this condition field equation becomes,

$$H^2 = -\frac{(m+2)^2}{9(2m+1)}\left(\frac{8\pi G}{3}\rho_M - \frac{f}{6} - \frac{Tf_T}{3}\right). \quad (3.14)$$

$$\left(H^2\right)' = \frac{m+2}{3}\left(\frac{2k^2 P_M + \frac{2(m+2)}{2m+1}Tf_T + \frac{4m+5}{2m+1}T + f}{2 + 2f_T + 4Tf_{TT}}\right). \quad (3.15)$$

where prime denotes the derivative with respect to $\ln\left(A^{\frac{m+2}{3}}\right)$.

[4] Polytropic Gas Dark Energy Cosmological Model:

Due to the high non-linearity of the field equations it is very difficult to evaluate the scale factors. So , in this case , we choose the scale factor of the form

$$B(t) = A^m(t) = a(t) = a_0(t_s - t)^{-h} \quad ,t \le t_s, \quad m \ge 2 \quad ,h > 0 \quad (4.1)$$

where n and s are positive real constants. For convenience , we take $s = 1$.

The equation of state of the polytropic gas is given by

$$p_\gamma = K\rho_\gamma^{1+1/n} \quad , \quad (4.2)$$

where $K > 0$ positive constant and n is the polytropic index.

For *T+f(T)=T* the above equations takes the form

$$H^2 = \frac{(m+2)^2}{9(2m+1)}\left[-\frac{8\pi G}{3}(\rho_M + \rho_{DE})\right]. \quad (4.3)$$

$$\left(H^2\right)' = \left(\frac{m+2}{3}\right)8\pi G\left[P_M + P_{DE} + \frac{4m+5}{3(2m+1)}(\rho_M + \rho_{DE})\right]. \quad (4.4)$$

Here we have assumed non-relativistic matter where pressure is zero i.e. $P_M = 0$. After comparing Eq.(3.14) with Eq.(3.16) and Eq.(3.15) with Eq.(3.17) the energy density and pressure of Dark Energy becomes

$$\rho_{DE} = \frac{-1}{16\pi G}\left(f + 2Tf_T\right). \quad (4.5)$$

$$P_{DE} = \frac{1}{16\pi G}\left(\frac{f - Tf_T - \frac{2(4m+5)}{2m+1}T^2 f_{TT}}{1 + f_T + 2Tf_{TT}}\right). \quad (4.6)$$

The energy conservation equations corresponding to standard matter and DE are

$$\dot{\rho}_M + 3H\rho_M = 0,$$

$$\dot{\rho}_{DE} + 3H\rho_{DE}(1 + \omega_{DE}) = 0 \quad (4.7)$$

Here $\omega_{DE} = \frac{P_{DE}}{\rho_{DE}}$ is the Equation of State parameter for $f(T)$ gravity.

The resulting Equation of State parameter is

$$\omega_{DE} = -\frac{\frac{f}{T} - f_T - \frac{2(4m+5)}{2m+1}Tf_{TT}}{\left(1 + f_T + 2Tf_{TT}\right)\left(\frac{f}{T} + 2f_T\right)} = -1 - \frac{1}{\frac{K}{B}\left[a_0\left(\frac{H}{h}\right)^h\right]^{3/n} - 1}, \quad h > 0 \quad (4.8)$$

Here, one should note that $\omega_{DE} < -1$ Which corresponds to a phantom accelerating universe.

[5] Conclusion

The Polytropic Gas Dark Energy model in *f(T)* gravity for Plane Symmetric universe has been studied. It is interesting to note that the Equation of State parameter ω_{DE} does not cross the phantom divide line. Hence, Polytropic Gas Dark Energy model in *f(T)* gravity for Plane Symmetric universe corresponds to a phantom accelerating universe. Therefore, the plane symmetric universe persists in the Dark Energy era.

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