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Modified Chaplygin Gas Dark Energy Model in *f (T)* Theory of Gravity

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Abstract:

The universe filled with Modified Chaplygin Gas Dark Energy has been studied in $f(T)$ theory of gravity. Detail study has been done with the help of equation of state parameter and phantom divide line .

Keywords: $f(T)$ *theory of gravity, Equation of state parameter, Modified Chaplygin gas , Dark energy.*

1] Introduction :

The most significantly studied modified theories of gravity are $f(R)$, $f(R,T)$, $f(T,\tau)$, $f(R,L_m)$ and so on. Ferraro and Fiorini (2007, 2008) developed *f(T)* theory of gravity for solving particle horizon problem and obtained singularity free solutions with positive cosmological constant. Bamba *et al*. (2011) studied the evolution of effective equation of state for different *f(T)* models as well as Daouda *et al*.(2012) examined this model with the holographic Dark Energy model to study the phantom behavior as well as unification of Dark Energy and Dark Matter. Plane symmetric Modified Chaplygin Gas Dark Energy model in *f(T)* theory of gravity has been studied in this article .

[2] $f(T)$ gravity & Plane Symmetric Solutions :

The modified field equations of the teleparallel theory of gravity (Sharif and Jawad 2012) are obtained by varying the action with respect to the vierbein vector field h_μ^i and are given by

$$\left[e^{-1}\partial_\mu\left(eS_i{}^{\mu\nu}\right)-h_i^\lambda T^\rho{}_{\mu\lambda}S_\rho{}^{\nu\mu}\right]\left(1+f(T)\right)+S_i{}^{\mu\nu}\partial_\mu(T)f_{TT}+\frac{1}{4}h_i^\nu\left[T+f(T)\right]=\frac{1}{2}k^2h_i^\rho T_\rho^\nu, \quad (1)$$

where $S_i^{\mu\nu}=h_i^\rho S_\rho^{\mu\nu}$, $k^2=8\pi G$, $f_T\equiv\frac{df}{dT}$.

The teleparallel Lagrangian density is described by the torsion scalar T and is defined as

$$T=S_\rho{}^{\mu\nu}T^\rho{}_{\mu\nu}, \quad (2)$$

where

$$S_\rho{}^{\mu\nu}=\frac{1}{2}\left(K^{\mu\nu}{}_\rho+\delta_\rho^\mu T^{\theta\nu}{}_\theta-\delta_\rho^\nu T^{\theta\mu}{}_\theta\right), \quad (3)$$

is the antisymmetric tensor.

Here, The torsion and contorsion tensors are defined as

$$T^\rho{}_{\mu\nu}=\Gamma^\rho{}_{\nu\mu}-\Gamma^\rho{}_{\mu\nu}=h_i^\rho(\partial_\mu h_\nu^i-\partial_\nu h_\mu^i), \quad (4)$$

$$K^{\mu\nu}{}_\rho=-\frac{1}{2}\left(T^{\mu\nu}{}_\rho-T^{\nu\mu}{}_\rho-T_\rho{}^{\mu\nu}\right). \quad (5)$$

The plane symmetric line element is in the form

$$ds^2=dt^2-R_1{}^2(t)(dx^2+dy^2)-R_2{}^2(t)dz^2, \quad (6)$$

where R_1 and R_2 are the cosmic scale factors.

Using above equations, the torsion T becomes

$$T=-2\left(2\frac{\dot{R}_1\dot{R}_2}{R_1R_2}+\frac{\dot{R}_1{}^2}{R_1{}^2}\right). \quad (7)$$

For above plane symmetric metric, the average scale factor a, the mean Hubble parameter H and the anisotropy parameter Δ of the expansion are respectively given by

$$a=(R_1{}^2R_2)^{\frac{1}{3}}, \quad (8)$$

$$H=\frac{1}{3}\left(2\frac{\dot{R}_1}{R_1}+\frac{\dot{R}_2}{R_2}\right), \quad (9)$$

$$\Delta=\frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i-H}{H}\right)^2, \quad (10)$$

where H_i are the directional Hubble paramerters in x, y and z direction respectively.

These are given by $H_1 = H_2 = \frac{\dot{R}_1}{R_1}$, $H_3 = \frac{\dot{R}_2}{R_2}$.

The energy momentum tensor for perfect fluid is given by

$$T_\rho^\nu = diag(\rho_M, -P_M, -P_M, -P_M), \quad (11)$$

where ρ_M is the energy density and P_M is the pressure of matter inside the Universe.

Using above, the field equations of *f(T)* gravity for $i = 0 = \nu$ and $i = 3 = \nu$ reduce to following set of equations:

$$T + f(T) - 4\left(2\frac{\dot{R}_1\dot{R}_2}{R_1R_2} + \frac{\dot{R}_1^{\,2}}{R_1^{\,2}}\right)(1 + f(T)) = 2k^2\rho_M, \quad (12)$$

$$4\left(\frac{\dot{R}_1\dot{R}_2}{R_1R_2} + \frac{\dot{R}_1^{\,2}}{R_1^{\,2}} + \frac{\ddot{R}_1^{\,2}}{R_1^{\,2}}\right)(1 + f(T)) - 16\frac{\dot{R}_1}{R_1}\left[\frac{\dot{R}_1}{R_1}\left(\frac{\ddot{R}_2^{\,2}}{R_2} - \frac{\dot{R}_2^{\,2}}{R_2^{\,2}}\right) + \left(\frac{\ddot{R}_1^{\,2}}{R_1} - \frac{\dot{R}_1^{\,2}}{R_1^{\,2}}\right)\left(\frac{\dot{R}_1}{R_1} + \frac{\dot{R}_2}{R_2}\right)\right] f_{TT} - (T + f) = 2k^2P_M$$

(13)

For solving the field equations (12) &(13), we consider that the expansion θ is proportional to shear σ. {Thorne 1967 suggests that from the observations of velocity red shift relation for extragalactic sources have shown that the isotropy of the Hubble expansion of the universe holds within about 30% range approximately (Kantowski and Sachs 1966) and further studies have given the limit to be $\frac{\sigma}{H} \le 0.30$, where σ is shear and H is Hubble constant. }.

For plane symmetric universe,the expansion scalar and shear scalar are given by

$$\theta = 2\frac{\dot{R}_1}{R_1} + \frac{\dot{R}_2}{R_2}, \quad (14)$$

$$\sigma = \frac{1}{\sqrt{3}}\left(\frac{\dot{R}_1}{R_1} - \frac{\dot{R}_2}{R_2}\right). \quad (15)$$

As per Bali and Kumavat (2008) , it leads to condition

$$R_2(t) = R_1^{\,m}(t), \quad m \ge 2. \quad (16)$$

Using this condition, we found

$$\Delta = 2\frac{(m-1)^2}{(m+2)^2}. \quad (17)$$

For Δ=0 , here , the isotropic behavior of the expanding Universe will be obtained.

Using such condition, the field equations reduce to

$$H^2 = -\frac{(m+2)^2}{9(m+1)}\left(\frac{8\pi G}{3}\rho_M - \frac{f}{6} - \frac{Tf_T}{3}\right). \quad (18)$$

$$(H^2)' = \frac{m+2}{3}\left(\frac{2k^2P_M + \frac{2(m+2)}{m+1}Tf_T + \frac{m+5}{m+1}T + f}{2 + 2f_T + 4Tf_{TT}}\right). \quad (19)$$

where prime denotes the derivative with respect to $\ln\left(A^{\frac{m+2}{3}}\right)$.

[4] Modified Chaplygin Gas Dark Energy Cosmological Model:

Here, we choose the scale factor of the form

$$R_2(t) = R_1^{\,m}(t) = a(t) = a_0(t_s - t)^{-h} \quad , t \le t_s, \quad m \ge 2 \quad , h > 0 \quad , \quad (20)$$

where n and s are positive real constants. For convenience , we take $s = 1$.

The equation of state of the Modified Chaplygin Gas Dark Energy model is given by

$$p_\gamma = A\rho_\gamma - \frac{B}{\rho_\gamma^\alpha}, \quad (21)$$

where $A > 0$; $B > 0$ are positive constants .

For *T+f(T)=T* , the above field equations reduce to

$$H^2 = \frac{(m+2)^2}{9(2m+1)}\left[-\frac{8\pi G}{3}(\rho_M + \rho_{DE})\right]. \quad (22)$$

$$(H^2)' = \left(\frac{m+2}{3}\right)8\pi G\left[P_M + P_{DE} + \frac{m+5}{(m+1)}(\rho_M + \rho_{DE})\right]. \quad (23)$$

Here we have assumed (for non-relativistic matter) $P_M = 0$.

Using above all equations, the energy density and pressure of Modified Chaplygin Gas Dark Energy becomes

$$\rho_T = \frac{-1}{16\pi G}(f + 2Tf_T). \quad (24)$$

$$P_T = \frac{1}{16\pi G}\left(\frac{f - Tf_T - \frac{(m+5)}{m+1}T^2 f_{TT}}{1 + f_T + 2Tf_{TT}}\right). \quad (25)$$

The energy conservation equations corresponding to Dark Matter and Dark Energy are

$$\dot{\rho}_M + 3H\rho_M = 0,$$

$$\dot{\rho}_{DE} + 3H\rho_{DE}(1+\omega_{DE}) = 0 \qquad (26)$$

Here $\omega_T = \dfrac{P_T}{\rho_T}$ is the Equation of State parameter for Modified Chaplygin Gas Dark Energy in $f(T)$ gravity.

The resulting Equation of State parameter for Modified Chaplygin Gas Dark Energy in $f(T)$ gravity will be given by

$$\omega_T = -\frac{\frac{f}{T} - f_T - \frac{(m+5)}{m+1}Tf_{TT}}{(1+f_T+Tf_{TT})\left(\frac{f}{T}+f_T\right)} = -1 + \frac{A+1}{\frac{B}{C(1+A)}\left[a_0\left(\frac{H}{h}\right)^h\right]^{3((1+\alpha)(1+A))}+1}, \quad h>0 \quad . \qquad (27)$$

For $C < 0$ and $\dfrac{B}{C(1+A)}\left[a_0\left(\dfrac{H}{h}\right)^h\right]^{3(1+\alpha)(1+A)} > 1$

ω_T can cross the phantom divide line.

5] Conclusion:

The Modified Chaplygin Gas Dark Energy Plane Symmetric universe has been studied in *f(T)* gravity . It is interesting to note that the Equation of State parameter ω_T can cross the phantom divide line in *f (T)* theory of gravity similar to Karami & Abdolmaleki (2010) .

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